Basic Properties of Electromagnetic Waves

Electromagnetic waves consist of oscillating electric and magnetic fields.

(site 1)

1. $\mathbf{E}(x, y, z, t) \perp \mathbf{B}(x, y, z, t)$

2. Both **E** and **B** are $\perp$ to the direction of wave motion. At any point in space at any time the direction of wave motion is parallel to $\mathbf{E} \times \mathbf{B}$

3. The wave speed in vacuum is $\mathrm{c} = 2.9979 \times 10^8 \mathrm{~m/s}$
Regardless of the wavelength or state of motion of the source or observer.

4. $|\mathbf{B}| = |\mathbf{E}|/c$ at each point in space at each time

The energy density in a light wave (energy per unit volume) is given by

$$u(x,y,z,t) = \varepsilon_0 \left|\mathbf{E}(x,y,z,t)\right|^2$$

where $\varepsilon_0 = 8.854 \times 10^{-12} \ \frac{\mathrm{J/m^3}}{(\mathrm{V/m})^2}$

This expression includes both the electric and magnetic field energy

Example:

An electromagnetic plane wave is moving in the +y direction. It’s electric field is polarized along the x-axis and has an amplitude of 10^3 V/m. The wavelength is 550nm.

A. Find an expression for the electric field

We know that

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{550\times 10^{-9}\, m} = 1.142\times 10^{7}\ \text{rad / m}$$

and that

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} = k\,c = \left(1.142\times 10^{7}\ \text{rad / m}\right)\left(3\times 10^{8}\ \text{m / s}\right)$$

$$= 3.426\times 10^{15}\ \text{rad / s}$$

so we can write

$$\mathbf{E}\,(\text{y}, \text{t}) = (10^{3}\,\tfrac{V}{m})\cos(k y - \omega t)\,\hat{\mathbf{x}}$$

B. Give an expression for the magnetic field of this wave

$$B = E/c = \frac{10^3 \ \frac{\mathrm{V}}{\mathrm{m}}}{3\times 10^8 \ \frac{m}{s}} = 3.33\times 10^{-6}\ \mathrm{T}$$

When **E** points along the $+x$- axis and the wave is moving in the $+y$ direction, we must have **B** pointing in the $-z$ direction. Therefore

$$\mathbf{B} = \left(3.33 \times 10^{-6}\ \mathrm{T}\right)\cos(ky - \omega t)(-\hat{\mathbf{z}})$$

$$= -\left(3.33 \times 10^{-6}\ \mathrm{T}\right)\cos(ky - \omega t)\,\hat{\mathbf{z}}$$

C. How much energy is contained in a cube of length 10nm centered at the location x = 1240 nm, y = 1180nm, z = 3412 nm at time t = $1.5 \times 10^{-15}\, s$?

The energy density is

$$u(x,y,z,t) = \varepsilon_0 \left|\mathbf{E}(x,y,z,t)\right|^2$$

$$= \left(8.854 \times 10^{-12} \frac{\text{J}/\text{m}^3}{(\text{V}/\text{m})^2}\right)\left(10^3 \frac{\text{V}}{\text{m}}\right)^2$$

$$\cos^2(1.142 \times 10^7 \tfrac{\text{rad}}{\text{m}} \cdot 1180 \times 10^{-9}\,\text{m} - 3.426 \times 10^{15} \tfrac{\text{rad}}{\text{s}} \cdot 1.5 \times 10^{-15}\,\text{s})$$

$$= 1.907 \times 10^{-3} \frac{\text{J}}{\text{m}^3}$$

so the total energy is u × volume =

$$1.907 \times 10^{-3} \frac{\text{J}}{\text{m}^3} \cdot \left(10^{-8}\,\text{m}\right)^3 = 1.907 \times 10^{-27}\,\text{J} = 1.190 \times 10^{-8}\,\text{eV}$$

Consider an electromagnetic wave whose electric field is given by $\mathbf{E} = \mathrm{E}_0 \cos(kx - \omega t)\,\hat{\mathbf{y}}$

The average energy density at any point in space is the average value of

$$u(x,y,z,t) = \varepsilon_0 \left|\mathbf{E}(x,y,z,t)\right|^2 = \varepsilon_0\, \mathrm{E}_0^2 \cos^2(kx - \omega t)$$

The average value of $\cos^2(\theta)$, averaged over one or more periods, is $\dfrac{1}{2}$

$$\left\langle \cos^2(\theta) \right\rangle = \frac{1}{2} \quad \text{so that} \quad \left\langle \cos^2(kx - \omega t) \right\rangle = \frac{1}{2}$$

$$\langle u \rangle = \varepsilon_0 \mathrm{E}_0^2 \left\langle \cos^2(kx - \omega t) \right\rangle = \tfrac{1}{2} \varepsilon_0 \mathrm{E}_0^2$$

Since the coordinates don't enter into this expression,

it holds for every electromagnetic plane wave.

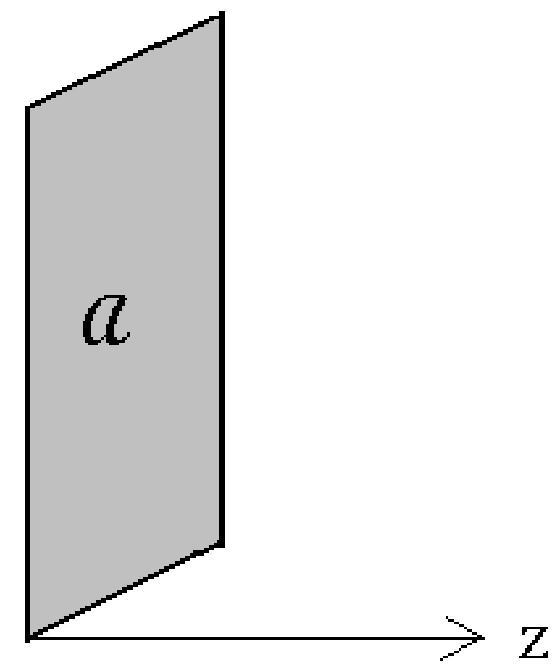
For example, for the wave discussed in the previous problem, the average energy density is

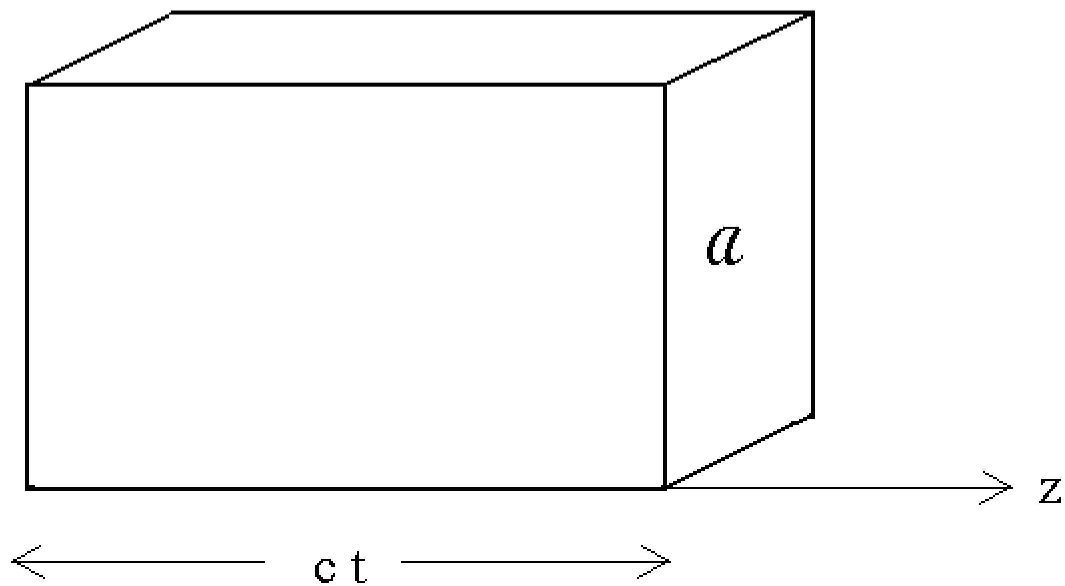
$$\langle u \rangle = \frac{1}{2} \cdot 8.854 \times 10^{-12} \frac{J/m^3}{(V/m)^2} \cdot \left(10^3 \frac{V}{m}\right)^2 = 4.427 \times 10^{-6} \frac{J}{m^3}$$

Consider an electromagnetic wave whose electric field is

$$\mathbf{E} = E_0 \cos(kz - \omega t)\,\mathbf{y}$$

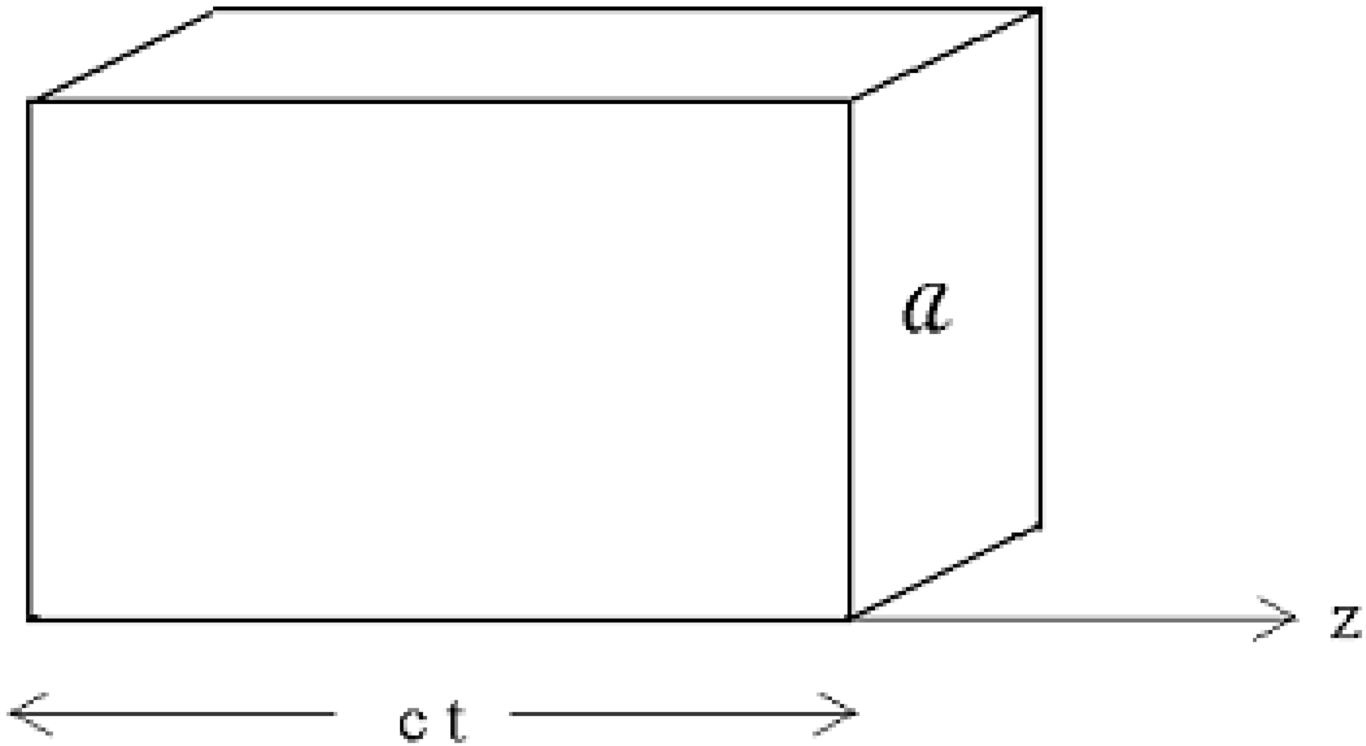
How much energy passes through a surface of area a perpendicular to the direction of wave motion in time t?




a
z
c t

The average energy in this box is

$$\text{average energy} = (\tfrac{1}{2}\varepsilon_0 E_0^2)\,\boldsymbol{a}\,ct$$



The rate at which energy passes through the surface is

$$P = \frac{\text{average energy}}{t} = \left(\tfrac{1}{2}\varepsilon_0 E_0^2\right)\boldsymbol{a}\, c$$

The average power per unit area is the intensity

$$I \equiv \frac{P}{\boldsymbol{a}} = \tfrac{1}{2}\varepsilon_0\, c\, E_0^2$$