M.SC. SEM.-III MATHEMATICS

Paper –I

Functional Analysis

EQUIVALENT NORMS

A norm $\|\cdot\|$ on a vector space X is said to be **equivalent** to a norm $\|\cdot\|_0$ on X if there are positive numbers a and b such that for all $x \in X$ we have

$$a\|x\|_0 \le \|x\| \le b\|x\|_0 \quad . \quad (1)$$

1.Lemma (linear combinations).

Let $\{x_1, \cdots, x_n\}$ be a linearly independent set of vectors in a normed space X (of any dimension) .Then there is a number $c > 0$ such that for any choice of scalars $\alpha_1, \cdots, \alpha_n$ we have

$$\|\alpha_1 x_1 + \cdots + \alpha_n x_n\| \geq c(|\alpha_1| + \cdots + |\alpha_n|) \quad (c > 0). \qquad (2)$$

Using Lemma 1 ,we can prove the following theorem.

2.Theorem.
On a finite dimensional vector space X ,any norm $\|\cdot\|$ is equivalent to any other norm $\|\cdot\|_o$.