

THE MAXIMIN – MINIMAX PRINCIPLE

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For player A , minimum value in each row represents the least gain (payoff) to him if he chooses his particular strategy. These are written in the matrix by row minima. He will then select the strategy that maximizes his minimum gains. This choice of player A is called the **maximin principle**, and the corresponding gain is called the maximin value of the game.

$$\text{maximin value} = v^*$$

For player B , on the other hand , likes to minimize his losses , the maximum value in each column represents the maximum loss to him if he chooses his particular strategy. These are written in the matrix by column maxima. He will then select the strategy that minimizes his maximum losses. This choice of player B is called the **minimax principle**, and the corresponding gain is called the minimax value of the game.

$$\text{minimax value} = v^{\wedge}$$

RULE FOR DETERMINING A SADDLE POINT

The procedure of determining saddle point is as follows :

- Step 1** : Select the minimum element of each row of the payoff matrix and mark them [*].
- Step 2** : Select the maximum element of each column of the payoff matrix and mark them [+].
- Step 3** : If there appears an element in the payoff matrix marked [*] and [+] both, the position of that element is a saddle point of the payoff matrix.
- REMARK** : 1) A game is said to be fair , if $v^{\wedge} = 0 = v^*$.
2) A game is said to be strictly determinable, if $v^{\wedge} = v = v^*$.

EXAMPLE ON SADDLE POINT

- ▶ Determine which of the following game is strictly determinable and fair.

1)

		player A	
		A ₁	A ₂
Player B	B ₁	5	0
	B ₂	0	2

Solution : the payoff matrix for player A is :

		player A		Row minima
		A ₁	A ₂	
Player B	B ₁	5 +	0*	0
	B ₂	0 *	2+	0
Column maxima		5	2	

EXAMPLE ON SADDLE POINT

The payoffs marked with [*] represent the minimum payoff in each row and those marked with [+] represent the maximum payoff in each column of the payoff matrix. The largest component of row minima represents v^{\wedge} (maximin value) and the smallest component of column maxima represents v^* (minimax value).

Thus obviously, we have

$$v^{\wedge} = 0 \text{ and } v^* = 2$$

Since $v^{\wedge} \neq v^*$, the game is not strictly determinable.

EXAMPLE ON SADDLE POINT

- Determine which of the following game is strictly determinable and fair.

2)

		player A	
		A ₁	A ₂
	B ₁	0	2
		Player B	
	B ₂	-1	4

Solution : the payoff matrix for player A is :

		player A		
		A ₁	A ₂	Row minima
	B ₁	0 *+	2	0
		Player B		
	B ₂	-1*	4+	-1
Column maxima		0	4	

EXAMPLE ON SADDLE POINT

The payoffs marked with [*] represent the minimum payoff in each row and those marked with [+] represent the maximum payoff in each column of the payoff matrix. The largest component of row minima represents v^\wedge (maximin value) and the smallest component of column maxima represents v^* (minimax value).

Thus obviously, we have

$$v^\wedge = 0 \text{ and } v^* = 0$$

As $v^\wedge \neq v^*$, the game is strictly determinable and fair.
Optimum strategies for player A and B are given by :

$$S_0 = (A_1, B_1).$$