# Rashtrasant Tukdoji Maharaj Nagpur university, Nagpur.

**Government Institute Of Science, Nagpur**

**Class**:- B.Sc.(sem-I)

**Subject**:-Mathematics

**paper II**:- Calculus

**Topic**:- Reduction formulae of Integration .

# Sub-topic:- Reduction formulae

A reduction formula connects an integral with another of the same type but of lower order. Thus a successive application of the reduction formula enables us to evaluate the given integral. Now we shall derive some standard form.

**Reduction formula for $\int \boldsymbol{sin^n}$xdx=$\frac{-sin^{n-1} cosx}{n}$ + $\frac{n-1}{n} \int \boldsymbol{sin^{n-2}}$xdx**

**Proof:-**

let $I_n = \int sin^n$ x dx= $\int sin^{n-1}$ x sinx dx

=$-sin^{n-1}\, cosx$-$\int (n-1) sin^{n-2}$xcosx(-cosx)dx

= $-sin^{n-1}\, cosx$-$\int (n-1) sin^{n-2}$xcosx(-cosx)dx

= $-sin^{n-1}\, cosx$+$\int (n-1) sin^{n-2}$x(1-$sin^2$)dx

= $-sin^{n-1}\, cosx$+$\int (n-1) sin^{n-2}$xdx- $(n-1) \int sin^n$ xdx

= $-sin^{n-1}\, cosx$+$\int (n-1) I_{n-2}$- $(n-1)\, I_n$

$I_n + (\text{n} - 1)I_n$=$-sin^{n-1}\, cosx$+$(n-1)I_{n-2}$

n$I_n$= $-sin^{n-1}\, cosx$+$(n-1)I_{n-2}$

$$I_n = \frac{-sin^{n-1} cosx}{n} + \frac{n-1}{n} I_{n-2}$$

$$\int sin^n \text{xdx} = \frac{-sin^{n-1} cosx}{n} + \frac{n-1}{n} \int sin^{n-2} \text{xdx}$$

Hence proved.

- **Reduction formula for** $\int \boldsymbol{cos^n}$**xdx=**$\frac{\boldsymbol{-cos^{n-1}\, sinx}}{\boldsymbol{n}}$ **+** $\frac{\boldsymbol{n-1}}{\boldsymbol{n}}\int \boldsymbol{cos^{n-2}}$**xdx**

**Proof:-**

let $I_n = \int cos^n$ xdx= $\int cos^{n-1}$ x cosx dx

= $cos^{n-1}\, xsinx$-$\int(n-1)cos^{n-2}$xsinx(-sinx)dx

=$cos^{n-1}\, sinx$-∫$(n-1)cos^{n-2}$x $sin^2$xdx

=$cos^{n-1}\, sinx$-∫$(n-1)cos^{n-2}$x(1-$cos^2$ )dx

=$cos^{n-1}\, sinx$+∫$(n-1)cos^{n-2}$xdx- $(n-1)\int cos^n$ xdx

= $cos^{n-1}\, sinx$+∫$(n-1)I_{n-2}$- $(n-1)\, I_n$

$I_n + (\text{n}-1)I_n$= $cos^{n-1}\, sinx$+$(n-1)I_{n-2}$

n$I_n$=$cos^{n-1}\, sinx$+$(n-1)I_{n-2}$

$I_n = \frac{cos^{n-1}\, sinx}{n}$ + $\frac{n-1}{n}I_{n-2}$

$\int sin^n$xdx=$\frac{-sin^{n-1}\, cosx}{n}$ + $\frac{n-1}{n}\int sin^{n-2}$xdx

Hence proved

- Evaluate $\int \boldsymbol{sin^4}$xdx

Solution:-

We have the reduction formula $\int sin^n$xdx=$\frac{-sin^{n-1} cosx}{n}$ + $\frac{n-1}{n}\int sin^{n-2}$xdx

Put n=4,2 successively.

$\int sin^4$xdx=$\frac{-sin^3 xcosx}{4}$ + $\frac{3}{4}\int sin^2$xdx.....(2)

$\int sin^2$xdx=$\frac{-sin\, xcosx}{2}$ + $\frac{1}{2}\int sin^0$xdx

But $\int sin^0$xdx=∫dx=x

$from\ (1) we\ get,$

$\int sin^4$xdx=$\frac{-sin^3 xcosx}{4}$ + $\frac{3}{4}$($\frac{-sin\, xcosx}{2}$ + $\frac{1}{2}$x)

=$\frac{-sin^3 xcosx}{4}$ - $\frac{3}{8}sinxcosx$+ $\frac{3}{8}$x+c

# Thank you