DUALITY IN LINEAR PROGRAMMING

INTRODUCTION

Associated with every L.P.P (maximization or minimization) there always exist another L.P.P which is based upon the same data and having the same solution. The original problem is called the **primal problem** while the associated one is called its **dual problem**.

The concept of duality is based on the fact that any L.P.P must be first put in its standard form before solving the problem by simplex method. Since, all the primal – dual computations are obtained directly from the simplex table , it is logical that we define the dual that may be constituent with the standard form of the primal.

GENERAL PRIMAL-DUAL PAIR

Standard primal problem :

Maximize $z = C_1X_1 + C_2X_2 + \dots\dots + C_nX_n$ subject to the constraints :

$a_{i1}x_1 + a_{i2}x_2 + \dots\dots\dots + a_{in}x_n = b_i$; $i = 1, 2, \dots\dots m$

$x_j \geq 0$; $j = 1, 2, \dots\dots n$

Dual problem :

Maximize $z^* = b_{1W1} + b_{2W2} + \dots\dots + b_{nWn}$ subject to the constraints :

$\mathbf{a_{1jW1} + a_{2jW2} + \dots\dots\dots + a_{mjWm} = b_i}$; $\mathbf{j = 1, 2, \dots\dots n}$

$_{Wi}$ (I = 1,2,……m) unrestricted

PRIMAL-DUAL PAIR IN MATRIX FORM

Standard primal problem :

Find $X^T \in R^n$ so as to maximize $z = cx$, $c \in R^n$

Subject to the constraints : $Ax = b$ and $x \geq 0$, $b^T \in R^m$

where A is an $m \times n$ matrix

Dual problem :

Find $w^T \in R^m$ so as to minimize $z^* = b^T w$ $b \in R^m$

Subject to the constraints : $A^T w \geq c^T$, $c \in R^n$

where is the transpose of an an $m \times n$ real matrix A and w is unresterícted in sign.

DUALITY THEOREMS

Theorem : The dual of the dual is primal.

Proof : Find X^T € R^n so as to maximize $f(x) = cx$, c € R^n

Subject to the constraints : $Ax = b$ and $x \geq 0$, b^T € R^m

where A is an $m \times n$ matrix

The dual of this primal is the L.P.P determining

w^T € R^m so as to minimize $f(w) = b^T w$ b € R^m

Subject to the constraints : $A^T w \geq c^T$, c € R^n

where is the transpose of an an $m \times n$ real matrix A and w is

unresitericted in sign

DUALITY THEOREMS

The standard form of dual then is to :

minimize $g(w) = b^T(w_1 - w_2)$, b € R^m

Subject to the constraints : $A^T(w_1 - w_2) - InS = c^T$, c € R^n

w_1 , w_2 , $S \geq 0$

Considering this L.P.P as our standard primal , the associated dual problem will be to maximize $h(y) = cy$, c € R^n

Subject to the constraints : $(A^T)^T \leq (b^T)^T$, $-(A^T)^T \leq -(b^T)^T$

$-y \leq 0$ ($y \geq 0$) and y is unrestricted.

DUALITY THEOREMS

Eliminating redundancy, the dual problrem may be re – written as :

maximize $h(y) = cy$, $c \in R^n$

Subject to the constraints :

$Ay \leq b$ and $Ay \geq b$ implies

$Ay = b$

$y \geq 0$

This problem which is the dual of the dual problem , is just the primal problem we had started with.

This completes the proof.