TOPIC:HERMITIAN AND SYMMETRIC KERNEL

Hermitian krenel: A kernel is said to be Hermitian if it has a following properties**,**

$$K(y,x)=\overline{K}(x,y)$$

Where bar denotes complex conjugate.

Symmetric kernel:A kernel is said to be Symmetric if it has a property

$$K(y,x)=K(x,y)$$

Remark: It should be noted that a real Hermitian kernel is a symmetric kernel

Throrem: If eigen values for Hermitian kernel exist then they are real.

Proof: Consider the integral equation

$$Ø(x) = \lambda \int K(x,y)\ Ø(y)dy \qquad \text{..........(1)}$$

Where λ is the eigen value associated with kernel K(x, y)

$$\overline{Ø}(x)=\overline{\lambda} \int \overline{K}\ (x,y)\ \overline{Ø}(y)dy \qquad \text{...............(2)}$$

We have seen that,

$$\int |Ø(x)|^2 dx=\int Ø(x)\ \overline{Ø}\ (y)dy$$

Using this,

$$\int |Ø(x)|^2 dx= \int \lambda \int K(x,y)\ Ø(y)dy\ \ \overline{Ø}(x)\ dx$$

$$= \lambda \iint K(x,y)\, \emptyset(y)\, \overline{\emptyset}(x)\, dy\, dx \quad \ldots\ldots\ldots\ldots(3)$$

Also,

$$\int |\emptyset(x)|^2 dx = \int \overline{\lambda} \int \overline{K}(x,y)\, \overline{\emptyset}(y) dy\, \emptyset(x)\, dx$$

By changing order of integration,

$$\int |\emptyset(x)|^2 dx = \overline{\lambda} \iint K(x,y)\, \emptyset(x)\, \overline{\emptyset}(y)\, dx\, dy$$

……..(K is hermitian kernel)

Now, Interchanging x and y , we get

$$\int |\emptyset(x)|^2 dx = \overline{\lambda} \iint K(x,y)\, \emptyset(y)\, \overline{\emptyset}(x)\, dy\, dx \quad \ldots\ldots\ldots\ldots(4)$$

From (3) and (4),

We get,

$$\lambda \iint K(x,y)\, \text{Ø}(y)\, \overline{\text{Ø}}(x)\, dy\, \mathrm{dx} = \overline{\lambda} \iint K(x,y)\, \text{Ø}(y)\, \overline{\text{Ø}}(x)\, dy\, \mathrm{dx}$$

But , $\iint K(x,y)\, \text{Ø}(y)\, \overline{\text{Ø}}(x)\, dy\, \mathrm{dx} \neq 0$

Theorefore, $\lambda = \overline{\lambda}$

λ *is real.*