Institute of Science,Nagpur

Class:B.scII   Maths paper v

Topic:

MEAN VALUE THEOREMS

ROLLE'S THEOREM

IF $,\boldsymbol{f(x)\ be\ a\ function\ such\ that}$

a. $f(x)$ is continuous in [a, b]

b. $f'(x)$ exists for every point in open interval (a,b)

c. $f(a)$= $f(b)$

Then there exists at least one point c ϵ (a, b) such that $f'^{(c)} = 0$

EX : Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$

Solution :

i.As $f(x) = x^2 - 6x+8$ is polynomial function, it continuous in [2,4]

ii. It is derivable in (2,4)

iii. $f(2)=f(4)=0$

So, by Rolle's theorem there must exist at least one number c between 2 and 4 such that $f'^{(c)} = 0$

now, $f'(x) = 2x-6$

Therefore, $f'(c) = 2c- 6 =0$ giver c = 3 and c ϵ (2,4)

therefore, theorem is verified.

LAGRANGE'S MEAN VALUE THEOREM

STATEMENT

IF $f(x)$ be a function such that

i. f $is\ continuous\ in\ closed\ interval\ [\ a\ ,b\ ]$
ii. f $is\ differentiable\ in\ (a,b)$

then there exists at least one point c ϵ (a,b) such that

$$\frac{f\ (b) - f\ (a)}{b\ - a} = f'(c)$$

EX. Verify Lagrange's mean value theorem for $f(x) = 2x^2 - 7x + 10 \ \ in \ \ [2, 5]$

Solution :

i. The function is polynomial function , hence continuous in [2,5]

ii. ii. It is derivable in (2,5) as $f'(x) = 4x - 7$ exists at each value of x in (2,5) and $f(a) = 4$, $f(b) = 25$

$$\text{Therefore , L.M.V.T. } \frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\text{or } \ 4c - 7 = \frac{25-4}{5-2} = 7$$

$$\text{Therefore , } 4c = 7+7 = 14$$

$$\text{Therefore } c = \frac{14}{4} = \frac{7}{2} \epsilon \ (2, 5)$$

Hence L.M.V.T . Is verified.

CAUCHY's MEAN VALUE THEOREM

STATEMENT

IF two functions $f(x)$ and $g(x)$ defined on [a,b] are

i. Continous on [a,b]

ii. Differntiable on (a,b) and

iii. $g(x) \neq 0$ *for any* $x \epsilon (a,b)$

then there exists at least one value c between a and b i.e. $c \; \epsilon \; (a,b)$ *such that*

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

Verify Cauchy Mean Value Theorem for $f(x) = e^{x}$, $ø(x) = e^{-x}$ in [a,b]

Solution :

$f(x) = e^{x}$, $\emptyset(x) = e^{-x}$,

f and∅ both are continuous in [a,b] , derivable in open interval (a,b)

Therefore , By Cauchy's theorem,

$$\frac{f(b)-f(a)}{\emptyset(b)-\emptyset(a)} = \frac{f'(c)}{\emptyset'(c)} \text{ or}$$

$$\frac{e^{b}-e^{a}}{e^{-b}-e^{-a}} = \frac{e^{c}}{-e^{-c}} \quad (\text{as } f'(x) = e^{x},\ ø(x) = -e^{-x})$$

$$\frac{e^{b}-e^{a}}{\frac{1}{e^{b}}-\frac{1}{e^{a}}} = -e^{2c} \quad \text{or} \quad (e^{b}-e^{a})\frac{e^{a}.e^{b}}{e^{a}-e^{b}} = -e^{2c}$$

$-e^{a+b} = -e^{2c}$ *therefore , a+ b = 2c*

Therefore $c = \frac{a+b}{2}$

Obiviouslyc= $\frac{a+b}{2} \epsilon\ (a,b)$.

Hence the theorem is verified