Subject:Integral Equations
Topic:Linear Integral equation.

- Definition: An Integral equation is said to be linear if the linear operation are performed in it upon the unkown function.
- General Form:Themost general form of linear integral equation is of the form, $v(x)u(x) = f(x) + \lambda \int_a k(x,t)u(t)dt$ .....(1)

Where,theupper limit may be either variables x or fixed . f, v, k are known function.Also,u is unkown function which is to be determine.λ is non-zero real or complex parameter and the kernel k(x,t)which is known function of integral equation.

Types of Linear Integral equation:

1)If v(x) is non zero

This is known as linear integral equation of 3rd kind.

2)If v(x)=1,then equation (1) becomes

$$u(x) = f(x) + \lambda \int_a k(x,t)u(t)dt$$

This is linear integral equation of 2nd kind.

3)If v(x)=0, then equation (1) becomes

$$0 = f(x) + \lambda \int_a k(x,t)u(t)dt$$

This is linaer integral equation of 1st kind.

- Classification of Linear Integral equation:

1)Fredholm Integral equation 2)VolteraIntegral equation

1)Fredholm integral equation:A linear integral equation of the form

$$v(x)u(x) = f(x) + \lambda \int_a^b k(x,t)u(t)dt$$

Where f, v, k are known function.Also,u is unkown function which is to be determine.λ is non-zero real or complex parameter and the kernel k(x,t)which is known function.this equation is called Fredholm integral equation.

- Special Cases:

1)Fredholm integral equation of 1st kind:

Put $v(x) = 0$,then

$$0= f(x) + \lambda \int_a^b k(x,t)u(t)dt$$

2)Fredholm integral equation of 2nd kind:

Put $v(x) = 1$,then

$$u(x) = f(x) + \lambda \int_a^b k(x,t)u(t)dt$$

3)Homogenous Fredholm integral equation f 2nd kind:

Put $f(x) = 0\ and\ v(x) = 1$,then

$$u(x) = \lambda \int_a^b k(x,t)u(t)dt$$

2) Voltera Integral Equation:A linear integral equation of the form,

$$v(x)u(x) = f(x) + \lambda \int_a^x k(x,t)u(t)dt$$

Where,a is constant.

f, v, k are known function.Also,u is unkown function which is to be determine.λ is non-zero real or complex parameter and the kernel k(x,t)which is known function.This equation is called Voltera integral equation.

- Special Cases;

1)Voltera integral equation of 1st kind:

Put $v(x) = 0$,then

$$0 = f(x) + \lambda \int_a^x k(x,t)u(t)dt$$

2)Voltera integral equation of 2nd kind:

Put $v(x) = 1$

$$u(x) = f(x) + \lambda \int_a^x k(x,t)u(t)dt$$

3) Voltera integral equation of 3rd kind:

Put $f(x) = 0 \; and \; v(x) = 1$ then,

$$u(x) = \lambda \int_a^x k(x,t)u(t)dt$$