TOPIC:Resolvent kernel for Fredholm Integral Equation.

Fredholm integral equation is

$$\emptyset(x) = f(x) + \lambda \int_a^b k(x,y)\emptyset(y)dy$$

has a solution of the form

$$\emptyset(x) = f(x) - \lambda \int_a^b R(x,y;\lambda)\emptyset(y)dy$$

Here Resolvent kernel is given by

$$R(x,y;\lambda) = -\sum_{n=1}^{\infty} \lambda^{n-1} K_n(x,y)$$

Example: Find the Resolvent kernel of K(x,t)=(1+x)(1-t). Here a=-1 and b=0.

Solution: By definition of itrated kernel, we have

$$K_1(x,t)=K(x,t)=(1+x)(1-t)$$

As $K_n(x,t)=\int_{-1}^{0} K(x,s)K_{n-1}(s,t)ds$

$$K_2(x,t)=\int_{-1}^{0} K(x,s)K_1(s,t)ds$$

$$=\int_{-1}^{0}(1+x)(1-s)(1+s)(1-t)ds$$

$$=(1+x)(1-t)\int_{-1}^{0}(1-s^2)ds$$

$$=(1+x)(1-t)\frac{2}{3}$$

Similarly, $K_3(x,t)=\int_{-1}^{0} K(x,s)K_2(s,t)ds$

$$=(1+x)(1-t)\frac{2}{3}\int_{-1}^{0}(1-s^2)ds$$

$$=(1+x)(1-t)\left(\frac{2}{3}\right)^2$$

In general, $K_n(x,t)=\left(\frac{2}{3}\right)^{n-1}(1+x)(1-t)$

$$R(x,y;\lambda)=-\sum_{n=1}^{\infty}\lambda^{n-1}K_n(x,y)$$

$$=-\sum_{n=1}^{\infty}\lambda^{n-1}\left(\frac{2}{3}\right)^{n-1}(1+x)(1-t)$$

$$=-(1+x)(1-t)\sum_{n=1}^{\infty}\lambda^{n-1}\left(\frac{2}{3}\right)^{n-1}$$