Rashtrsant Tukdoji Maharaj Nagpur university, Nagpur

GOVERNMENT INSTITUTE OF SCIENCE, NAGPUR

CLASS:- B.SC.(SEM-I)

SUBJECT:-MATHEMATICS

PAPER II:- CALCULUS

TOPIC:- PARTIAL DIFFERENTIATION

Sub topic:-Partial Differentiation

- Definition:- let z=f (x,y) the function of two independent variable x and y. If we keep y constant and x varies then z becomes a function of x only. The derivative of z with respect to x, keeping y as constant is called partial derivative pf z w. r. to.x and it is denoted by the symbols $\frac{\partial z}{\partial x}, \frac{\partial f}{\partial x}$ $etc.$
- $\frac{\partial z}{\partial x} = \lim_{\partial x \to 0} \frac{f(x+\partial x, y) - f(x,y)}{\partial \mathrm{x}}$
- The derivative of z with respect to y, keeping x as constant is called partial derivative pf z w. r. to.y and it is denoted by the symbols $\frac{\partial z}{\partial y}, \frac{\partial f}{\partial y}$ $etc.$
- $\frac{\partial z}{\partial y} = \lim_{\partial x \to 0} \frac{f(x, y+\partial y) - f(x,y)}{\partial \mathrm{y}}$
- This process of finding derivative of a function of two or more variables with respect to one of these variables keeping others constant is called as **partial differentiation.**
- $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2}, \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2}, \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x \partial y}$,etc.

Example 1:-z(x+y)=$x^2 + y^2$,show that $(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y})^2 = 4(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y})$

SOLUTION:- GIVEN Z(X+Y)=$x^2 + y^2 \Rightarrow$ Z=$\frac{x^2+y^2}{X+Y}$, THEN

$\frac{\partial z}{\partial x} = \frac{(x+y)2xy - x^2 + y^2}{(x+y)^2}$

$\frac{\partial z}{\partial x} = \frac{2xy + x^2 - y^2}{(x+y)^2}$

SIMILARLY, $\frac{\partial z}{\partial y} = \frac{2xy - x^2 + y^2}{(x+y)^2}$

NOW, $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = [\frac{2(x^2+y^2)}{X+Y}]^2 = 4[\frac{(x-y)}{X+Y}]^2 \ldots\ldots\ldots (1)$

AND $\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) = [1 - \frac{2xy + x^2 - y^2}{(x+y)^2} - \frac{2xy - x^2 + y^2}{(x+y)^2}]$

$= \frac{2xy + x^2 - y^2 - 4xy}{(x+y)^2} = [\frac{(x-y)}{X+Y}]^2$

$4(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}) = 4 \ldots\ldots\ldots\ldots (2)$

From (1) and (2)

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

Hence Proved .

FOR EXAMPLE 2:- U= $\frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2 + y^2 + z^2$ IS NOT EQUAL TO ZERO.

$$\frac{\partial u}{\partial x^2}^2 + \frac{\partial u}{\partial y^2}^2 + \frac{\partial u}{\partial z^2}^2 = 0$$

Thank you