B.Sc. Sem.-I
Mathematics

Paper-I

Algebra & Trigonometry

Hyperbolic Functions

Analogous to circular sine and cosine functions, we can define hyperbolic sine and cosine functions as follows:

$$\cosh \mathrm{x} = \frac{\mathrm{e}^{\mathrm{x}} + e^{-x}}{2}, \quad \sinh \mathrm{x} = \frac{\mathrm{e}^{\mathrm{x}} - e^{-x}}{2}$$

$$\tanh \mathrm{x} = \frac{\sinh \mathrm{x}}{\cosh \mathrm{x}} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\coth \mathrm{x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}, \operatorname{sec} h \ \mathrm{x} = \frac{2}{\mathrm{e}^{\mathrm{x}} + e^{-x}},$$

$$\cos ech \ \mathrm{x} = \frac{2}{\mathrm{e}^{\mathrm{x}} - e^{-x}}.$$

Relation between hyperbolic and circular functions

$$(1)\ \cos ix = \cosh x \quad \text{so} \quad \cosh ix = \cos x$$

$$(2)\ \sin ix = i \sinh x \quad \text{so} \quad \sinh ix = i \sin x.$$

$$(3)\ \tan ix = i \tanh x$$

$$(4)\ \cot ix = -i \coth x$$

$$(5)\ \sec ix = \operatorname{sech} x$$

$$(6)\ \operatorname{cosec} ix = i \operatorname{cosech} x$$

Formulae for hyperbolic functions

$$(i)\ \cosh^2 x - \sinh^2 x = 1$$

$$(ii)\ \cosh^2 x + \sinh^2 x = \cosh 2x$$

$$(iii)\ \sinh 2x = 2\sinh x \cosh x$$

$$(iv)\ \sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$