Govt. Institute of Science
Department of Mathematics
M.Sc. Sem- I
Topology-I

$\boldsymbol{T_0 - Space}$: $A\ topological\ space\ (X, \tau) is\ a\ T_0 - Space$
$iff\ for\ every\ pair\ (x, y)\ of\ distinct\ elements\ x, y \epsilon X$,
$\exists\ an\ open\ set\ which\ contains\ one\ of\ them\ but\ not\ the\ other.$

Hereditary Property : A property of a topological space which is preserved by its subspace is called hereditary property.

Topological Property : A property of a topological space which is preserved by its homeomorphic image is called topological property.

Example: Prove that a property of being a $T_0 - Space$ is both Hereditary as well as Topological.

Solution: Let (X, τ) be a $T_0 - Space$.

To prove hereditary property:

Let (X^*, τ^*) be a subspace of (X, τ).

We have to prove X^* is also $T_0 - Space$.

Let $x, y \epsilon X^*$ such that $x \neq y$

Then $x, y \epsilon X$ such that $\neq y$, Since $X^* \subseteq X$

As X is $T_0 - Space$, $\exists$ an open set G in X, such that $x \in G$ but $y \notin G$

Thus, we can find an open set G^* in subspace topo for X^* such that $G^* = X^* \cap G$ and $x \in G^*$ but $y \notin G^*$.

This shows that X^* is $T_0 - Space$.

Hence a property of being a $T_0 - Space$ is hereditary.

To prove Topological Property:

Let (X, τ) be a $T_0 - Space$ and (X^*, τ^*) be a arbitrary topological space.

Let $f: X \to X^*$ be homeomorphism from $T_0 - Space\ X$ onto an arbitrary topological space X^*.

We have to prove $f(X) = X^*$ is also $T_0 - Space$.

Let $x, y \epsilon X$ such that $x \neq y$

Then $f(x)$ & $f(y)$ are two distinct pts in X^*.

As X is $T_0 - Space$, $\exists$ an open set G in X, such that $x \in G$ but $y \notin G$.

But, a mapping $f: X \to X^*$ is a homeomorphism and therefore the image $f(G)$ is an open set in X^* such that $f(x) \in f(G)\ but\ f(y) \notin f(G)$. Thus $f(X) = X^*$ is also $T_0 - Space$.

Hence a property of being a $T_0 - Space$ is topological.

$T_1 - Space$:

A topological space (X, τ) is a $T_1 - Space$ iff for every pair (x, y) of distinct elements $x, y \epsilon X$, $\exists$ two open sets, one containing x but not y, and other containing y but not x.

Example: Prove that a property of being a $T_1 - Space$ is both Hereditary as well as Topological.

Example: Prove that every $T_1 - Space$ is $T_0 - Space$, but converse is not true.